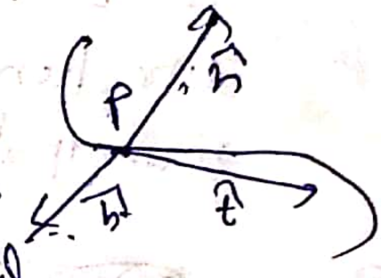
Serret - Frenet Formulae

~~Let r = r(u) be a space curve.~~

Let C be space curve defined by $\vec{r}(u)$.
If u is taken along s. (s = arc length) then $\frac{d\vec{r}}{ds}$ is a unit tangent vector and denoted $\hat{t}$.



The rate at which $\hat{t}$ changes w.r.t. s is a measure of curvature of C is and is given by $\frac{d\hat{t}}{ds}$. If $\hat{n}$ is the unit vector in the direction $\frac{d\hat{t}}{ds}$ then it is called normal or principal normal. Then $\frac{d\hat{t}}{ds} = k\hat{n}$ where k is called curvature and $1/k$ is called radius of curvature. If ~~B~~ $\hat{b}$ is perpendicular to $\hat{t}$ and $\hat{n}$ i.e. $\hat{b} = \hat{t} \times \hat{n}$ then $\hat{b}$ is called binormal.

The Serret - Frenet formulae ~~is~~ are given by

$$\frac{d\hat{t}}{ds} = k\hat{n}$$

$$\frac{d\hat{n}}{ds} = -k\hat{t} + \tau\hat{b}$$

$$\frac{d\hat{b}}{ds} = -\tau\hat{n}$$

Where $\hat{t}$ ~~two~~, $\hat{n}$, $\hat{b}$ ~~respectively~~ are unit tangent, normal, bino

κ is the curvature and τ is the torsion.

- Osculating Plane: The plane containing tangent and principal normal at P
- Normal Plane: The plane containing ~~t~~ normal and binormal at any point P.
- Rectifying Plane: The plane containing tangent and binormal.

Proof of Serret - Frenet formulae

① $\boxed{\dfrac{d\hat{t}}{ds} = \kappa\hat{n}}$

We know that $\hat{t}.\hat{t} = 1$

$\Rightarrow \dfrac{d\hat{t}}{ds}.\hat{t} = 0$. Therefore $\dfrac{d\hat{t}}{ds}$ perpendicular to $\hat{t}$. If $\hat{n}$ is the unit ~~be~~ vector in the direction $\dfrac{d\hat{t}}{ds}$ then $\dfrac{d\hat{t}}{ds} = \kappa\hat{n}$. $\hat{n}$ is called the principal normal and κ is curvature.

② $\cancel{\hat{b} = \hat{t} \times \hat{n}}$

$\cancel{\dfrac{d\hat{b}}{ds} = \dfrac{d\hat{t}}{ds}}$

② $\hat{b} = \hat{t} \times \hat{n}$

$$\frac{d\hat{b}}{ds} = \frac{d\hat{t}}{ds} \times \hat{n} + \hat{t} \times \frac{d\hat{n}}{ds}$$

$$= \kappa\hat{n}\times\hat{n} + \hat{t} \times (-\kappa\hat{t} + \tau\hat{b})$$

$$= \tau\,\hat{t}\times\hat{b} = \tau(-\hat{n}) = -\tau\hat{n}$$

$$\boxed{\frac{d\hat{b}}{ds} = -\tau\hat{n}}$$

(3) $\hat{n} = \hat{b} \times \hat{t}$

$$\frac{d\hat{n}}{ds} = \frac{d\hat{b}}{ds} \times \hat{t} + \hat{b} \times \frac{d\hat{t}}{ds}$$

$$= (-\tau \hat{n}) \times \hat{t} + \hat{b} \times \kappa \hat{n}$$

$$= +\tau \hat{b} - \kappa \hat{t}.$$

$$\boxed{\frac{d\hat{n}}{ds} = \tau \hat{b} - \kappa \hat{t}}$$